

DIGITAL FILTER

Cross-Reference to Related Application

This application claims the priority of Provisional Application U.S. Serial No. 60/148,787, filed August 13, 1999, and entitled DIGITAL FILTER, which is incorporated herein
5 by reference.

Background of the Invention

This application relates to sound systems for lifelike reproduction of digitally stored sound, e.g., music. It is well known that "live" sound contains recognizable harmonics which are lost during electronic processing and storage. It is also known that "canned" music may be harmonically enhanced during playback by passing the audio output signal through an appropriately designed circuit. Suitable circuits for this purpose are disclosed in U.S. Patent 5,361,306 and in U.S. Ser. No. 08/472,876, filed June 7, 1995, and entitled APPARATUS AND METHOD OF ENHANCING ELECTRONIC AUDIO SIGNALS; U.S. Ser. No. 08/700,728, - 381198 Mei
10 filed August 13, 1996, and entitled APPARATUS AND METHODS FOR THE HARMONIC Enhancement
15 ENHANCEMENT OF ELECTRONIC AUDIO SIGNALS; U.S. Ser. No. 08/909,807, filed 381161 Pendleton
August 12, 1997, and entitled APPARATUS AND METHODS FOR THE HARMONIC Pendleton
ENHANCEMENT OF ELECTRONIC AUDIO SIGNALS; and U.S. Ser. No. 08/989,373, filed 381161 Lee
December 12, 1997, and entitled APPARATUS AND METHODS FOR ENHANCING 2644
ELECTRONIC AUDIO SIGNALS; each of which is owned by the assignee of the present
20 application and incorporated herein by reference. PK2-8

Special problems arise when sound is stored on a digital recording medium (e.g., a compact disc). The creation of the digital recording typically involves conversion of the live sound into an audio signal in analog form. The amplitude of the audio signal is then sampled at regularly spaced intervals and digitized for storage. On playback the sound system generates an
25 output signal at the sampling frequency. The sampling frequency conforms to industry standards, most commonly 44,100 Hz. That is only slightly more than twice the highest frequency which

may be heard by the unaided human ear and is therefore near the theoretical lower limit for full bandwidth recording (i.e., the Nyquist critical frequency). Tonal components required for high quality sound reproduction can be easily lost or severely limited by compromises and constraints placed upon the recording process.

- 5 There is a need for a digital device able to provide high quality sound reproduction such as in a personal computer environment.

Summary of the Invention

10 A digital device is provided for simply and inexpensively enhancing digital sound samples from a compact disk (CD) or other digital information storage medium in such a way that the quality of audible sound produced from the sound samples more closely approaches that of the sound heard live in an acoustically designed environment.

15 In accordance with a first embodiment of the present invention, a digital filter is provided comprising a series of digitized time coefficients stored in a memory. The time coefficients are mapped to a like number of frequency coefficients. The frequency coefficients are spaced at frequency intervals, have either zero phase angles or linearly spaced phase angles and have amplitudes which are mirrored about a mid frequency to produce periodicity of a time response for the digital filter.

20 The time coefficients may be even or odd in number. Preferably, they are odd integers greater than or equal to 5, preferably 7, more preferably 9 and most preferably 11. The time coefficients are defined by inverse discrete Fourier transforms of the frequency coefficients.

 In one embodiment of the present invention, a portion of the frequency coefficients having frequencies within a free band are selected so as to achieve a generally constant oscillation frequency across a center band which is broader than the free band.

 The frequency coefficients are spaced at equal frequency intervals.

25 In accordance with a second aspect of the present invention, a method of making a digital filter is provided and comprises the step of establishing a plurality of frequency response

coefficients separated at frequency intervals. The frequency response coefficients are graphically characterized by a base point, a series of principal points and a series of mirror points and have either zero phase angles or linearly spaced phase angles. The base point has a frequency of zero. A portion of the principal points are situated at frequencies encompassing the range of human hearing and a portion of the mirror points have frequencies and amplitudes which mirror the portion of the principal points when viewed relative to a mid frequency higher than the frequencies of the principal points. The method further comprises the steps of performing inverse discrete Fourier transformations to map the frequency response coefficients into corresponding time response coefficients, and storing the time response coefficients in a digital memory.

Preferably, the portion of the principal points are situated at predetermined frequencies within the range of human hearing and have amplitudes that roughly are inversely corresponding to human hearing sensitivity at the predetermined frequencies.

The frequency response coefficients are established at uniformly spaced frequency intervals.

The establishing and performing steps preferably comprise the following steps:
selecting a plurality of first frequency response coefficients separated at uniformly spaced frequency intervals, wherein the first frequency response coefficients have either zero phase angles or linearly spaced phase angles and each first frequency response coefficient further has an amplitude and a frequency;

arranging the plurality of first frequency response coefficients in order from lowest frequency to highest frequency to define a list of first frequency response coefficients;

performing inverse discrete Fourier transformations to map the plurality of first frequency response coefficients into corresponding first time response coefficients;

discarding a pair of the first time response coefficients which have equal magnitudes and are positioned adjacent to one another in the list, with remaining time response coefficients defining second time response coefficients;

assessing the effect on a frequency response of the digital filter after discarding the pair of the first time response coefficients;

repeating the performing, discarding and assessing steps until a pair of discarded time response coefficients cause a significant change in the frequency response of the digital filter; and

adding to remaining time response coefficients the pair of discarded time response coefficients causing a significant change in the frequency response of the digital filter. The added and remaining time response coefficients comprise final time response coefficients.

The method of making a digital filter preferably further comprises the steps of: multiplying each of the final time response coefficients by an integer conversion number to define converted final time response coefficients, wherein the conversion number is sufficiently large to permit discarding any remaining fractional portion without losing substantial final time response coefficient accuracy; and discarding from each of the converted final time response coefficients any remaining fractional portion. The conversion number is preferably selected as a power of two.

The assessing step preferably comprises the steps of: generating a first frequency response curve from the first frequency response coefficients; performing discrete Fourier transformations to map the second time response coefficients into corresponding second frequency response coefficients; generating a second frequency response curve from the second frequency response coefficients; and comparing the first and second frequency response curves to determine if the second frequency response curve is substantially different from the first frequency response curve.

Each of the frequency response coefficients has an amplitude and a frequency. The range of human hearing is within a band of frequencies having a low end and a high end. Preferably, a portion of the frequency response coefficients having frequencies between a reference frequency and the high end increase in amplitude as per increasing frequencies from the reference frequency toward the high end. It is also preferred that a portion of the frequency response coefficients

having frequencies between a reference frequency and the low end increase in amplitude as per decreasing frequencies from the reference frequency toward the low end.

In one embodiment of the present invention, the frequency response coefficients having frequencies between the reference frequency and the high end increase in amplitude up to a significant amplitude peak at a peak high frequency and decrease in amplitude as per increasing frequencies toward the high end above the peak high frequency.

In another embodiment of the present invention, the frequency response coefficients having frequencies between the reference frequency and the high end increase in amplitude up to a significant amplitude peak at a peak high frequency, decrease in amplitude as per increasing frequencies down to a significant amplitude trough at a trough high frequency and increase in amplitude as per increasing frequencies toward the high end.

The reference frequency may fall within a range of from about 501 Hz to about 8018 Hz. The peak high frequency may fall within a range of from about 2004 Hz to about 20045 Hz. The amplification of the frequency response coefficient at the peak high frequency may be from about 1.3 times to about 6.0 times the amplification of the frequency response coefficient at the reference frequency.

In accordance with a third aspect of the present invention, a method is provided for enhancing a series of digital audio samples comprising the steps of: receiving the series of digital audio samples, and generating a driving signal by convolving the series of samples in real time with a series of stored time coefficients. The time coefficients are mapped to a like number of frequency coefficients. The frequency coefficients are spaced at frequency intervals, have either zero phase angles or linearly spaced phase angles, and have amplitudes which are mirrored about a mid frequency.

The method for enhancing a series of digital audio samples may further comprise the step of generating an analog audio signal from the driving signal.

The time coefficients are preferably integer time coefficients.

The step of generating a driving signal comprises the step of:

repeatedly solving the following equation for Y:

$$Y(n) = A_0X(n) + A_1X(n-1) + \dots + A_{N-1}X(n-[N-1])$$

where A_0 through A_{N-1} are the stored time coefficients;

$X(n)$ is the most recent sample received;

$X(n-1)$ through $X(n-[N-1])$ correspond to $N-1$ samples received prior to sample $X(n)$;

n is the running index of the time coefficients being computed;

N is equal to the number of terms in the equation to the right side of the equal sign; and

wherein calculated values of Y define the driving signal.

The step of generating a driving signal may further comprise the steps of dividing the values of Y by a number previously used to convert initial real number time coefficients to integer time coefficients; and discarding any remaining fractional portion of the divided values of Y .

The receiving step may comprise the step of reading the series of digital samples from a digital recording medium or from a compressed file. The receiving step may also comprise the step of downloading audio sample streams from the Internet.

In accordance with a fourth aspect of the present invention, an apparatus is provided for enhancing a series of digital audio samples. The apparatus comprises a device for receiving the series of digital samples and a digital filter comprising a series of stored time response coefficients. The time response coefficients are mapped to a like number of frequency response coefficients. The frequency response coefficients are spaced at frequency intervals, have phase angles of zero and have amplitudes which are mirrored about a mid frequency. The apparatus further comprises a microprocessor for generating a driving signal by convolving the sound

samples in real time against the time coefficients.

The receiving device comprises a digital signal reader.

The microprocessor convolves the sound samples in real time against the time response coefficients by repeatedly solving the following equation for Y:

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$$Y(n) = A_0X(n) + A_1X(n-1) + \dots + A_{N-1}X(n-[N-1])$$

where A_0 through A_{N-1} are the stored time coefficients;

$X(n)$ is the most recent sample received;

$X(n-1)$ through $X(n-[N-1])$ correspond to N-1 samples received prior to sample
10 $X(n)$;

n is the running index of the time coefficients being computed;

N is equal to the number of terms in the equation to the right side of the equal
sign; and

wherein calculated values of Y define the driving signal.

15 The microprocessor may further divide the values of Y by a number previously used to convert real number time coefficients to integer time coefficients and discards any remaining fractional portion of the divided values of Y.

The apparatus may further comprise a converting device responsive to the driving signal for generating an analog audio signal from the driving signal.

20 In accordance with a fifth aspect of the present invention, a filter package is provided having two or more parallel digital filters. A first filter is provided which comprises a series of digitized first time coefficients stored in a first memory. The time coefficients are mapped to a like number of first frequency coefficients. The first frequency coefficients have either zero phase angles or linearly spaced phase angles and have amplitudes which are mirrored about a mid
25 frequency. A second digital filter is provided and comprises a series of digitized second time

coefficients at least one of which has a value which is different from each of the first time coefficients. The second time coefficients are stored in a second memory and mapped to a like number of second frequency coefficients. The second frequency coefficients have either zero phase angles or linearly spaced phase angles and have amplitudes which are mirrored about a mid frequency.

Preferably, the number of the first time coefficients is equal to or greater than 5 and the number of the second time coefficients is equal to or greater than 5. The first memory and the second memory may comprise the same memory component.

In accordance with a sixth embodiment of the present invention, an apparatus is provided for enhancing a series of digital audio samples. The apparatus comprises a device for receiving the series of digital samples and a filter package. The filter package has a first digital filter comprising a series of digitized first time response coefficients stored in a first memory. The time coefficients are mapped to a like number of first frequency coefficients. The first frequency coefficients have either zero phase angles or linearly spaced phase angles and have amplitudes which are mirrored about a mid frequency. The filter package also includes a second digital filter comprising a series of digitized second time response coefficients at least one of which has a value which is different from each of the first time coefficients. The second time coefficients are stored in a second memory and mapped to a like number of second frequency coefficients. The second frequency coefficients have either zero phase angles or linearly spaced phase angles and have amplitudes which are mirrored about a mid frequency. The apparatus further includes a microprocessor for generating a driving signal by convolving the sound samples in real time against either the first time response coefficients or the second time response coefficients.

The apparatus may further comprise an input device coupled to the microprocessor for selecting one of the first filter and the second filter. The microprocessor generates the driving signal by convolving the sound samples in real time against the first time response coefficients when the first filter is selected and the second time response coefficients when the second filter is

selected. The apparatus may also comprise a converting device responsive to the driving signal for generating an analog audio signal from the driving signal.

Brief Description of the Drawings

Fig. 1 is a block diagram of one embodiment of an audio playback system according to the invention.

Fig. 2 is a schematic illustration of a frequency response curve for a digital filter in a first embodiment.

Fig 3 is a schematic illustration of a frequency response curve for a digital filter in a second embodiment.

Fig. 4 is a schematic illustration of a frequency response curve for a digital filter in a third embodiment.

Fig. 5 is a schematic illustration of a frequency response curve for a digital filter in a fourth embodiment.

Fig. 6 is a schematic illustration of a frequency response curve for a digital filter in a fifth embodiment.

Fig. 7 is a schematic illustration of a frequency response curve for a digital filter in a sixth embodiment.

Detailed Description

Fig. 1 illustrates a personal computer 10, organized in accordance with the present invention. In the illustrated arrangement, personal computer 10 has a microprocessor 20, a CD reader 30 and a sound card 40. There also is a digital filter 50 which preferably comprises an array of storage locations 52 within a random access memory. Further portions of the memory (not illustrated) may be devoted to other uses. A series of N time response values, A_0 through A_{N-1} , are tabulated in a written program for downloading into storage locations 52. These

response values, hereinafter referred to as time coefficients, are established by a technique described below.

A system clock (not illustrated) causes CD reader 30 to read a series of digitized sound samples, at a predetermined sampling frequency such as 44,100 Hz. Many other sampling rates exist. For example, sampling rates of 22,050 Hz and 11,025 Hz are used in many PC applications. Some audio CD's are now produced using other sampling rates. Digital video disks are produced at sampling rates of 64,000 Hz. The telephone industry samples at a rate of 8000 Hz. The apparatus and methods set out herein are applicable to any sampling rate.

Each sample is designated $X(n)$ at the time of sampling and is stored temporarily in a designated storage location 22. The sound samples are later shifted successively through a series of $N-1$ other storage locations 22 for generation of time shifted samples $X(n-1)$ through $X(n-[N-1])$. Storage locations 22, may be thought of as being equivalent to stages of a shift register within microprocessor 20 and are so illustrated in Fig. 1. However, microprocessor 20 may not have sufficient memory for this purpose, and therefore the computer program may cause the sound samples to be shifted through a designated portion of the random access memory.

Microprocessor 20 multiplies each A_n by a corresponding $X(n)$, as indicated by the multiplication blocks 24. The $A_n X(n)$ products then are summed as indicated by a set of summation blocks 26. It will be appreciated that microprocessor 20 performs N shift operations, N multiplications and N summations for each sound sample read by CD reader 30. The calculated sums are supplied to sound card 40 at 44,100 Hz and define a driving signal. Operations within CD reader 30 and sound card 40 are effected by conventional calls to the operating system. Shifting of samples, multiplication by time response coefficients and summation is a matter of routine programming and need not be further described.

The control program for personal computer 10 convolves N time response coefficients $A_0 - A_{N-1}$ against the entire series of sound samples. Mathematically speaking, the computer repeatedly solves the following equation:

$$Y(n) = A_0X(n) + A_1X(n-1) + \dots + A_{N-1}X(n-[N-1])$$

where A_0 through A_{N-1} are the stored time coefficients;

$X(n)$ is the most recent sample received;

$X(n-1)$ through $X(n-[N-1])$ correspond to $N-1$ samples received prior to sample

5 $X(n)$;

n is the running index of the time coefficients being computed;

N is equal to the number of terms in the equation to the right side of the equal sign; and

wherein calculated values of Y define the driving signal.

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When sampling at 44,100 Hz, all calculations implied by the above equation must be completed within a sampling interval, Δ , of 22.68 microseconds. Actually much less time is available, because the operating system requires time for performing its normal housekeeping functions and running any other applications which simultaneously may be open. Consequently it is necessary to find a series of discrete values for the time response function which are few in number, yet able to produce the high quality sound reproduction desired. In accordance with a preferred embodiment of this invention it has been found that digital music of the desired quality may be generated for an odd value of N (the number of time response coefficients) as low as 7. A noticeable degree of enhancement of digital music has even been accomplished with a value of N as low as 5. Accordingly, it has been found that the value of N is at least 5, preferably 7, more preferably 9, and most preferably 11. It is also contemplated that the value of N may comprise an odd integer greater than 11. It is further contemplated that the value of N may comprise an even integer greater than or equal to 6, such as 8, 10 or 12. However, it is preferred that N comprise an odd integer.

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As described above, the convolution process is carried out in real time using appropriate time response coefficients. These time response coefficients are calculated off-line and are mapped to a like number of frequency coefficients established as discussed below. That procedure generates N frequency coefficients H_0-H_{N-1} , each having a frequency and an amplitude,

which are mapped into N time response coefficients A_0 - A_{N-1} by use of the discrete inverse Fourier transformation:

$$A_n = \frac{1}{N} \sum_{k=0}^{N-1} H_k e^{+i2\pi kn/N}$$

By way of example, N may have a value of 19, which would call for 19 time response coefficients, all derived from a set of 19 frequency coefficients, regularly spaced at intervals F_s in accordance with the time-frequency uncertainty principle, a relationship stated in the following equation:

$$F_s = \frac{1}{\Delta N}$$

Thus for $\Delta = 1 / 44,100$ and $N = 19$, $F_s = 2321$ Hz. The results are set forth in TABLE I below.

TABLE 1

| <u>Freq. ~ Hz</u> | <u>Amp of Freq Coeff</u> | <u>Amp of Time Coeff</u> | <u>Phase Angle of Time Coeff</u> |
|-------------------|--------------------------|--------------------------|----------------------------------|
| 0 | 2.5 | 3.0295 | 0 |
| 2321 | 2.5 | 0.4046 | Π |
| 4642 | 2.15 | 0.01 | Π |
| 6963 | 2.4 | 0.1905 | 0 |
| 9284 | 2.6 | 0.0237 | Π |
| 11605 | 2.9 | 0.0694 | Π |
| 13926 | 3.78 | 0.2137 | 0 |
| 16247 | 4.6 | 0.2074 | Π |
| 18568 | 2.6 | 0.1175 | 0 |
| 20889 | 4 | 0.0714 | Π |

| | | | | |
|---|-------|------|--------|-------|
| | 23210 | 4 | 0.0714 | Π |
| | 25531 | 2.6 | 0.1175 | 0 |
| | 27852 | 4.6 | 0.2074 | Π |
| | 30173 | 3.78 | 0.2137 | 0 |
| 5 | 32494 | 2.9 | 0.0694 | Π |
| | 34815 | 2.6 | 0.0237 | Π |
| | 37136 | 2.4 | 0.1905 | 0 |
| | 39457 | 2.15 | 0.01 | Π |
| | 41778 | 2.5 | 0.4046 | Π |

In general the A_n and the H_k are all complex numbers, having a magnitude and a phase. However, in the practice of a first embodiment of this invention the H_k have a phase angle of 0, and the A_n have a phase angle of either 0 or Π . Values of Π (i.e., 3.14159) are accommodated in the filter design by assigning a negative value to the amplitude of the time coefficient. The 19 frequency domain points define a frequency response curve 58 extending from 0 Hz to 41,778 Hz as illustrated in Fig. 2. They include a base point 60, principal points 61 - 69 and mirror points 71 - 79. Frequency domain points 60 - 79 are set at regular intervals of 2321 Hz with mirror points 71 - 79 having amplitudes equal to principal points 61 - 69 respectively. It will be observed that the principal points and the mirror points are mirrored about a mid frequency of 22,050 Hz, half the 44,100 Hz sampling frequency. This produces a periodic response at the sampling frequency. That periodicity effectively extends the frequency response curve 58 to 44,100 Hz and sets a virtual point (not illustrated) at 44,100 Hz having a magnitude equal to that of base point 60. This virtual frequency domain point is mapped to a virtual time response point (also not illustrated) equal to, and synchronous with, A_0 . Since 19 is an odd number there is no frequency domain point at the mirroring mid frequency.

Since the human ear does not respond to frequencies in excess of about 20,000 Hz, the entire information content of the original audio signal can be recorded by low pass filtering at 22,050 Hz, followed by sampling and recording at 44,100 Hz. CDs and other digital recordings

are usually prepared in this fashion. That gives two points per cycle of the highest frequency present.

Frequency response points were selected in the range from 20 Hz to 20,000 Hz having amplitudes bearing a roughly inverse relationship to the sensitivity of the human ear over that same range. This flattens out the perceived response of the filter to the regenerated sound. In particular the frequency response was relatively decreased at points 68 and 62 to compensate for regions of increased ear sensitivity near 4642 Hz and 18,568 Hz respectively, and it was relatively increased at point 63 to compensate for reduced ear sensitivity in the region around 16,247 Hz. Thus, as is apparent from Fig. 2, frequency response coefficients 62-69 having frequencies between a reference frequency of 4642 Hz and a high end frequency of 20,000 Hz increase in amplitude as per increasing frequencies from the reference frequency up to a significant amplitude peak at a peak high frequency of 16,247 Hz and decrease in amplitude as per increasing frequencies down to a significant amplitude trough at a trough high frequency of 18,568 Hz. The amplification of the frequency response coefficient at the peak high frequency is about 2.1 times the amplification of the frequency response coefficient at the reference frequency. The frequency response coefficient 69 which is positioned between the reference frequency and a low end frequency of 20 Hz has an amplitude which is slightly greater than that of the frequency response coefficient 68 located at the reference frequency.

The required filter response is undefined outside the range of human hearing. That includes the region below about 20 Hz, a free band region between 20,000 Hz and 24,100 Hz, and an end region between 24,100 Hz and 44,100 Hz. However the periodic nature of the filtering process requires some definition for all frequencies between zero and the sampling frequency. Therefore, frequency response points were selected at amplitudes which would tend to lend smoothness to the response function in the undefined regions. Hence, point 60 having a frequency of 0 Hz was selected so as to have an amplitude of 2.5, which is equal to the amplitude for point 69, which has a frequency of 2321 Hz. The amplitudes of points 61 and 71, which fall within the free band region, were selected so as to achieve a generally constant oscillation

frequency across a center band extending from about 11,605 Hz to about 32,494 Hz, i.e., the portions of the curve defined between points 65,62; 62,72; and 72,75 have approximately the same width. Points 72-79 are mirror images of points 62-69. By selecting the amplitudes of points 60, 61 and 71-79 in the manner discussed above, the number of time coefficients having significant or large magnitudes were reduced. In order to reduce processing time during filtering, the number of time coefficients incorporated into the filter must be minimized. By minimizing the number of time coefficients having significant magnitudes, remaining time coefficients having insignificant magnitudes may be discarded resulting in fewer time coefficients required to achieve a desired frequency response for the filter.

The above described digital filter design procedure was found to produce time response coefficients which, when convolved with typical stored music, rendered output sound of remarkably high quality. It was found that background sound which is ordinarily suppressed during storage, was substantially restored upon playback. Moreover, 19 coefficients did not overwhelm the computer. Having achieved those results, filters having lower values of N were then designed.

A digital filter in accordance with a second embodiment of the present invention was created as follows. A set of first frequency response coefficients separated at uniformly spaced frequency intervals were selected. The first frequency response coefficients had zero phase angles. In the illustrated embodiment, the frequency response coefficients set out in Table 1 above were used as the first frequency response coefficients. By the use of discrete inverse Fourier transformation, the plurality of first frequency response coefficients were mapped into corresponding first time response coefficients, which are also set out in Table 1.

As noted above, in order to reduce processing time during filtering, the number of time coefficients should be minimized. So as to achieve that end, a pair of the first time response coefficients having equal magnitudes and being positioned adjacent to one another in Table 1 were removed. The discarded pair of time coefficients each had a magnitude of .0714. The remaining time coefficients defined second time coefficients.

If an even number of first time response coefficients are provided, the one first time response coefficient not having a zero frequency and not having a mate of equal magnitude is also discarded.

The effect of removing the pair of time coefficients on the frequency response of the digital filter was then assessed. This involved generating a first frequency response curve from the first frequency response coefficients. The second time response coefficients were mapped into corresponding second frequency response coefficients by use of the discrete Fourier transformation:

$$H_k = \sum_{n=0}^{N-1} A_n e^{-j \frac{2\pi kn}{N}}$$

A second frequency response curve was then generated using the second frequency response coefficients. The first and second frequency response curves were compared to determine if the second frequency response curve was substantially different from the first frequency response curve. If not, then another pair of time response coefficients, i.e., a pair of the second time response coefficients, having equal magnitudes and being positioned adjacent to one another were removed. This process of removing and assessing continued until a pair of discarded time response coefficients caused a significant change in the perceived enhancement caused by the frequency response of the digital filter. A significant change in the perceived enhancement caused by the frequency response of the digital filter corresponds to a substantial difference between an initial and a subsequent frequency response curve. When a significant change occurred, this last pair of discarded time response coefficients were added back to the time response coefficients. These remaining time response coefficients defined final time response coefficients.

In order to reduce processing time, it is preferred that the time response coefficients comprise integer numbers. In order to convert the final time response coefficients into integers,

they were multiplied by an integer conversion number sufficiently large to permit any remaining fractional portion to be discarded without losing substantial final time response coefficient accuracy. A substantial loss in final time response coefficient accuracy occurs when the resulting frequency response of the digital filter produces an enhancement perceptibly different from the desired enhancement. It is also preferred that the integer conversion number be selected as a power of two so that shifting may be used in place of division when subsequent renormalization occurs for calculated values of Y. In the illustrated embodiment, the final time coefficients (set out in brackets in Table 2 below) were multiplied by 2^{13} (8192). Any remaining fractional portions of the converted time coefficients were discarded. The integer final time coefficients are set out in Table 2 below. They may be tabulated in a written program for downloading into storage locations 52. This filter has 11 time response coefficients.

Prior to the microprocessor 20 supplying the calculated values of Y to the sound card 40, the microprocessor 20 must renormalize those values. This involves dividing each value of Y by the integer conversion number. Alternatively, if the integer conversion number is selected as a power of two, the microprocessor 20 can effect renormalization by right shifting an appropriate number of bit positions, 13 in the illustrated embodiment.

A frequency response curve 80 plotted from the final frequency response coefficients, set out in Table 2, is shown in Fig. 3. Curve 80 has a base point 100. It also has principal points 101 - 105, which are mirrored by points 111- 115. As is apparent from Fig. 3, frequency response coefficients 102-105 having frequencies between a reference frequency of 4009 Hz and a high end frequency of 20,000 Hz increase in amplitude as per increasing frequencies from the reference frequency toward the high end.

TABLE 2

| <u>Freq. ~ Hz</u> | <u>Amp of Freq Coeff</u> | <u>Amp of Time Coeff</u> | <u>Phase of Time Coeff</u> |
|-------------------|--------------------------|--------------------------|----------------------------|
| 00000 | 2.50 | 23,980 (2.9273) | 0 |
| 04009 | 2.00 | 3,593 (.4386) | Π |

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| | | | | | |
|---|-------|------|-------|----------|---|
| | 08018 | 2.40 | 125 | (0.0153) | 0 |
| | 12027 | 2.85 | 1226 | (0.1497) | 0 |
| | 16036 | 4.00 | 598 | (0.0731) | Π |
| | 20045 | 3.50 | 1089 | (0.1330) | 0 |
| 5 | 24054 | 3.50 | 1089 | (0.1330) | 0 |
| | 28063 | 4.00 | 598 | (0.0731) | Π |
| | 32072 | 2.85 | 1226 | (0.1497) | 0 |
| | 36081 | 2.40 | 125 | (0.0153) | 0 |
| | 40090 | 2.00 | 3,593 | (0.4386) | Π |

10 A filter in accordance with a third embodiment of the present invention was created using the same design procedures undertaken to construct the filter of the second embodiment. The third filter's final frequency and time response coefficients are set out in Table 3 below. The final time response coefficients both before and after being multiplied by an integer conversion number equal to 8192 are set out in Table 3. A frequency response curve 90 plotted using the final frequency response coefficients is shown in Fig. 4. The curve has a base point 120, 15 principal points 121-125 and mirror points 131-135. As is apparent from Fig. 4, frequency response coefficients 122-125 having frequencies between a reference frequency of 4009 Hz and a high end frequency of 20,000 Hz increase in amplitude as per increasing frequencies from the reference frequency toward the high end.

TABLE 3

| | <u>Freq. ~ Hz</u> | <u>Amp of Freq Coeff</u> | <u>Amp of Time Coeff</u> | <u>Phase of Time Coeff</u> |
|----|-------------------|--------------------------|--------------------------|----------------------------|
| | 00000 | 2.50 | 24,948 (3.0455) | 0 |
| | 04009 | 1.00 | 5958 (0.7274) | Π |
| 5 | 08018 | 2.50 | 1634 (0.1995) | Π |
| | 12027 | 3.50 | 3094 (0.3777) | 0 |
| | 16036 | 5.00 | 237 (0.0290) | Π |
| | 20045 | 3.50 | 2503 (0.3056) | 0 |
| | 24054 | 3.50 | 2503 (0.3056) | 0 |
| 10 | 28063 | 5.00 | 237 (0.0290) | Π |
| | 32072 | 3.50 | 3094 (0.3777) | 0 |
| | 36081 | 2.50 | 1634 (0.1995) | Π |
| | 40090 | 1.00 | 5958 (0.7274) | Π |

The above description of the invention teaches a straight forward calculation of the sound card driving signal, Y. A more preferred procedure takes advantage of an observed symmetry in the time coefficients. That is, $A_m = A_n$ where $0 < n \leq (N + 1) / 2$ and $m = N - n$. In the case where $N = 11$ a simplified calculation of the sound card driving signal takes the form:

$$Y = A_0 X_0 + A_1 (X_1 + X_{10}) + A_2 (X_2 + X_9) + A_3 (X_3 + X_8) + A_4 (X_4 + X_7) + A_5 (X_5 + X_6)$$

This reduces the number of time consuming multiplications. The procedure may be extended to other values of N.

Final frequency and time response coefficients for filters of fourth, fifth and sixth embodiments of the present invention are set out in Tables 4-6 below. The final time response coefficients both before and after being multiplied by an integer conversion number equal to 8192 are set out in Tables 4-6. Nine frequency and time response coefficients are provided for the filter set out in Table 4; seven frequency and time response coefficients are provided for the

filter set out in Table 5; and five frequency and time response coefficients are provided for the filter set out in Table 6. A frequency response curve 140 for the fourth filter is shown in Fig. 5, a frequency response curve 150 for the fifth filter is shown in Fig. 6, and a frequency response curve 160 for the sixth filter is shown in Fig. 7. The fourth and fifth filters provided an enhancement which is comparable to that provided by the first, second and third filters set out in Tables 1-3. While the enhancement of the sixth filter was not equal to that of the fourth and fifth filters, a noticeable enhancement did occur.

TABLE 4

| <u>Freq. ~ Hz</u> | <u>Amp of Freq Coeff</u> | <u>Amp of Time Coeff</u> | <u>Phase of Time Coeff</u> |
|-------------------|--------------------------|--------------------------|----------------------------|
| 0 | 2 | 24,576 (3.0000) | 0 |
| 4900 | 1 | 7175 (0.8759) | II |
| 9800 | 3 | 361 (0.0441) | II |
| 14700 | 4 | 1365 (0.1667) | 0 |
| 19600 | 4.5 | 2075 (0.2533) | 0 |
| 24500 | 4.5 | 2075 (0.2533) | 0 |
| 29400 | 4 | 1365 (0.1667) | 0 |
| 34300 | 3 | 361 (0.0441) | II |
| 39200 | 1 | 7175 (0.8759) | II |

TABLE 5

| <u>Freq. ~ Hz</u> | <u>Amp of Freq Coeff</u> | <u>Amp of Time Coeff</u> | <u>Phase of Time Coeff</u> |
|-------------------|--------------------------|--------------------------|----------------------------|
| 0 | 2 | 21,064 (2.5714) | 0 |
| 6300 | 1 | 6198 (0.7566) | II |
| 12600 | 3 | 1330 (0.1624) | 0 |
| 18900 | 4 | 2526 (0.3084) | 0 |
| 25200 | 4 | 2526 (0.3084) | 0 |

| | | | |
|-------|---|---------------|----|
| 31500 | 3 | 1330 (0.1624) | 0 |
| 37800 | 1 | 6198 (0.7566) | II |

TABLE 6

| <u>Freq. ~ Hz</u> | <u>Amp of Freq Coeff</u> | <u>Amp of Time Coeff</u> | <u>Phase of Time Coeff</u> |
|-------------------|--------------------------|--------------------------|----------------------------|
| 0 | 2 | 16,384 (2.000) | 0 |
| 8820 | 1 | 3663 (0.4472) | II |
| 17640 | 3 | 3663 (0.4472) | 0 |
| 26460 | 3 | 3663 (0.4472) | 0 |
| 35280 | 1 | 3663 (0.4472) | II |

Filters constructed in accordance with the present invention may have a reference frequency which falls within the range of from about 501 Hz to about 8018 Hz; a peak high frequency which falls within the range of from about 1002 Hz to about 20045 Hz; a trough high frequency which can fall at any frequency after the peak high frequency; and a peak low frequency which falls within the range of from about 0 Hz to about 2004 Hz. It is further contemplated that the amplification of the frequency response coefficient at the peak high frequency may be about 1.3 times to about 6.0 times the amplification of the frequency response coefficient at the reference frequency. It is also contemplated that the amplification of the frequency response coefficient at the peak low frequency may be about 1.1 times to about 3.0 times the amplification of the frequency response coefficient at the reference frequency.

It is also contemplated that filters of the present invention, including the filters set out herein may be used with a microprocessor or like device coupled to a digital signal reader (e.g., a real audio editor, CD or DVD player, etc.) which reads digital signals. The source of such digital signals can be digital samples stored on a digital recording medium as well as digital signals from, for example, audio streams (i.e., compressed packets of digital information) received over the Internet; MP3, liquid audio, a2b, and other compressed files received over the Internet and

stored on any conventional memory device; etc. When uncompressed data is read, the audio signals may be processed directly. When compressed data is read, the data must be uncompressed prior to processing. Instead of sending the driving signal directly to a sound card, it is contemplated that a microprocessor, after generating the driving signal or Y-values, may store the driving signal in any conventional memory device capable of storing digital data, transfer the driving signal over the Internet, the airways (via for example radio or television signals, microwaves, etc.), a network, cable TV lines, the telephone lines, etc. to another digital signal reader, or digital signal storing device.

Frequency response coefficients for a filter of a seventh embodiment of the present invention are set out in Table 6A below. The frequency response coefficients have linearly spaced phase angles. This is in contrast to the zero phase angles of the time response coefficients set out in Tables 1-6 above. The frequency response coefficient may be mapped into corresponding time response coefficients by use of the discrete inverse Fourier transformation set out above.

TABLE 6A

| <u>Freq. ~ Hz</u> | <u>Amp of Freq. Coeff</u> | <u>Phase Angle of Freq. Coeff</u> |
|-------------------|---------------------------|-----------------------------------|
| 0 | 2.5 | 0 |
| 4009 | 1 | -0.57 |
| 8018 | 2.5 | -1.14 |
| 12027 | 3.5 | -1.71 |
| 16036 | 5 | -2.28 |
| 20045 | 3.5 | -2.85 |
| 24054 | 3.5 | -3.42 |
| 28063 | 5 | -3.99 |
| 32072 | 3.5 | -4.56 |
| 36081 | 2.5 | -5.13 |

| | | |
|-------|-----|-------|
| 40090 | 1 | -5.7 |
| 44099 | 2.5 | -6.27 |

It is additionally contemplated that two or more of the filters described herein may be stored in a memory component as a filter package such that a user can select any one of the filters making up the package for use in enhancing digital audio samples. For example, a filter package may comprise a first filter having a series of digitized first time coefficients stored in the memory component. The first time coefficients are mapped to a like number of first frequency coefficients. The first frequency coefficients have either zero phase angles or linearly spaced phase angles and have amplitudes which are mirrored about a mid frequency. The filter package further includes a second digital filter comprising a series of digitized second time coefficients at least one of which has a value which is different from each of the first time coefficients. The second time coefficients are stored in a second memory and mapped to a like number of second frequency coefficients. The second frequency coefficients have either zero phase angles or linearly spaced phase angles and have amplitudes which are mirrored about a mid frequency.

An input device (not shown) such as a keyboard is coupled to a microprocessor for allowing the user to select one of the first and second filters for use at any given time for enhancement of digital audio samples. The microprocessor generates a driving signal by convolving the sound samples in real time against the first time response coefficients when the first filter is selected and the second time response coefficients when the second filter is selected. The apparatus may also comprise a converting device responsive to the driving signal for generating an analog audio signal from the driving signal.

It is contemplated that the filter package may be provided to a prospective customer who intends to purchase a hardware version of a digital filter. Each filter of the software filter package is then tested by the customer. Once a desired filter is selected, the filter manufacturer designs a hardware version of the selected filter and provides it to the customer for testing. Accordingly, the digital filter package allows a filter manufacturer to provide two or more filters

to a prospective customer for testing without having to design and build two or more separate hardware filters.

It is contemplated that the time coefficients set out in Table 1 may be multiplied by an integer conversion number.

5 The digital filters disclosed above are finite impulse response (FIR) filters. The procedure for designing an infinite impulse response (IIR) filter implementation can be very similar to the above described FIR procedure. Assuming that a desired frequency response has been specified at various frequencies, an IIR filter can be developed using the same technique described above for the FIR filter implementation.

10 Using a conventional IIR filter coefficient computation technique, such as that found in Matlab (a software system for filter design and digital signal processing), choose an IIR filter order and input the desired filter order and frequency response. The Matlab function will return the IIR filter coefficients that meet the frequency response and filter order requirements, within some tolerance that is a function of the chosen filter order. Note that the IIR filter has the form (first-order example), as follows:

$$y(k) = b_0 * x(k) + a_1 * y(k-1)$$

15 so that there are feedback terms involving the output. The FIR includes terms from the input, i.e., $x(k)$.

20 The filter coefficients are prepared for real-time implementation as in the FIR case. Each coefficient is multiplied by a large number and the decimal portion of the result is eliminated. In this way the coefficients are made integers, which can make the real-time implementation faster for most computer systems. Because improper scaling could lead to instability, care should be taken when performing this operation with an IIR filter due to the feedback term.

25 The IIR filter has the advantage of potentially requiring fewer coefficients and therefore a more efficient real-time implementation. This means that an IIR filter may take less time to compute an output sample than an equivalent FIR filter. A possible disadvantage of the IIR filter is that it typically does not possess linear phase. If the filter design requires a linear phase, as is preferred for

the frequency response described above, then an FIR filter is normally the only choice. The IIR filter may have approximately linear phase and may only approximate the magnitude response, depending on the order of the IIR filter specified in the design. It is possible to obtain linear phase from an IIR filter by running the data through the IIR filter twice (both forward and backward). However, this doubles the order of the filter.

The follow equation is a 3rd order IIR filter approximation of the frequency response disclosed above.

$$y(k) = 0.0355x(k) - 0.0709x(k-1) + 0.1509x(k-2) + 0.0533x(k-3) \\ - 0.2944y(k-1) - 0.0399y(k-2) + 0.1033y(k-3)$$

Upon comparison, there will likely be some difference between the original FIR frequency response and the 3rd order IIR filter frequency response. Although they are similar in amplitude and shape, they may create slightly noticeable differences in music quality. In addition, the phase response for the 3rd order IIR filter is nonlinear, especially at the higher frequencies, although it appears to be very linear up to about 15 kHz. Hence, at higher frequencies the nonlinear phase may also contribute to noticeable differences in reproduction quality.

Final frequency and time response coefficients for other FIR digital filters of the present invention are set out below in Tables 7- 34. The final time response coefficients have not yet been multiplied by an integer conversion factor.

TABLE 7

| Freq | Amplitude | Time Coefficient | Phase |
|-------|-----------|------------------|--------|
| 0 | 2 | 2.7273 | 0 |
| 4009 | 1 | 0.6916 | 3.1416 |
| 8018 | 2 | 0.1595 | 3.1416 |
| 12027 | 3.5 | 0.2665 | 0 |
| 16036 | 4 | 0.1129 | 0 |
| 20045 | 3.5 | 0.1081 | 0 |
| 24054 | 3.5 | 0.1081 | 0 |
| 28063 | 4 | 0.1129 | 0 |
| 32072 | 3.5 | 0.2665 | 0 |
| 36081 | 2 | 0.1595 | 3.1416 |
| 40090 | 1 | 0.6916 | 3.1416 |

Phase means phase of the time coefficient. All Frequency phases are zero.

TABLE 8

| Freq | Amplitude | Time Coefficient | Phase |
|-------|-----------|------------------|--------|
| 0 | 1.72 | 1.8709 | 0 |
| 4009 | 1 | 0.4524 | 3.1416 |
| 8018 | 1.35 | 0.2061 | 0 |
| 12027 | 1.65 | 0.0067 | 3.1416 |
| 16036 | 2.28 | 0.0949 | 0 |
| 20045 | 3.15 | 0.0826 | 0 |
| 24054 | 3.15 | 0.0826 | 0 |
| 28063 | 2.28 | 0.0949 | 0 |
| 32072 | 1.65 | 0.0067 | 3.1416 |
| 36081 | 1.35 | 0.2061 | 0 |
| 40090 | 1 | 0.4524 | 3.1416 |

Phase means phase of the time coefficient. All Frequency phases are zero.

TABLE 9

| Freq | Amplitude | Time Coeff | Phase |
|-------|-----------|------------|--------|
| 0 | 2.5 | 3.1091 | 0 |
| 4009 | 2 | 0.5577 | 3.1416 |
| 8018 | 2.5 | 0.0106 | 3.1416 |
| 12027 | 2.85 | 0.3027 | 0 |
| 16036 | 5 | 0.2475 | 3.1416 |
| 20045 | 3.5 | 0.2085 | 0 |
| 24054 | 3.5 | 0.2085 | 0 |
| 28063 | 5 | 0.2475 | 3.1416 |
| 32072 | 2.85 | 0.3027 | 0 |
| 36081 | 2.5 | 0.0106 | 3.1416 |
| 40090 | 2 | 0.5577 | 3.1416 |

Phase means phase of the time coefficient. All Frequency phases are zero.

TABLE 10

| Freq | Amplitude | Time Coefficient | Phase |
|-------|-----------|------------------|--------|
| 0 | 2.5 | 2.9273 | 0 |
| 4009 | 1 | 0.7106 | 3.1416 |
| 8018 | 2.5 | 0.0861 | 3.1416 |
| 12027 | 2.85 | 0.3286 | 0 |
| 16036 | 5 | 0.1285 | 3.1416 |
| 20045 | 3.5 | 0.383 | 0 |
| 24054 | 3.5 | 0.383 | 0 |
| 28063 | 5 | 0.1285 | 3.1416 |
| 32072 | 2.85 | 0.3286 | 0 |
| 36081 | 2.5 | 0.0861 | 3.1416 |
| 40090 | 1 | 0.7106 | 3.1416 |

Phase means phase of the time coefficient. All Frequency phases are zero.

TABLE 11

| Freq | Amplitude |
|-------|-----------|
| 0 | 2.1 |
| 2321 | 2 |
| 4642 | 1 |
| 6963 | 1 |
| 9284 | 1 |
| 11605 | 1 |
| 13926 | 1 |
| 16247 | 1 |
| 18568 | 1 |
| 20889 | 1 |
| 23210 | 1 |
| 25531 | 1 |
| 27852 | 1 |
| 30173 | 1 |
| 32494 | 1 |
| 34815 | 1 |
| 37136 | 1 |
| 39457 | 1 |
| 41778 | 2 |

TABLE 12

| Freq | Amplitude |
|--------|-----------|
| 0 | 1.6 |
| 21050 | 1.5 |
| 42100 | 1 |
| 63150 | 1.12 |
| 84200 | 1.2 |
| 105250 | 1.3 |
| 126300 | 1.4 |
| 147350 | 1.6 |
| 168400 | 2 |
| 189450 | 1.9 |
| 210500 | 1.9 |
| 231550 | 2 |
| 252600 | 1.6 |
| 273650 | 1.4 |
| 294700 | 1.3 |
| 315750 | 1.2 |
| 336800 | 1.12 |
| 357850 | 1 |
| 378900 | 1.5 |

TABLE 13

| Freq | Amplitude |
|-------|-----------|
| 0 | 1.51 |
| 1764 | 1.72 |
| 3528 | 1 |
| 5292 | 1.15 |
| 7056 | 1.21 |
| 8820 | 1.36 |
| 10584 | 1.41 |
| 12348 | 1.65 |
| 14112 | 1.98 |
| 15876 | 2.12 |
| 17640 | 2.49 |
| 19404 | 2.31 |
| 21168 | 2.11 |
| 22932 | 2.11 |
| 24696 | 2.31 |
| 26460 | 2.49 |
| 28224 | 2.12 |
| 29988 | 1.98 |
| 31752 | 1.65 |
| 33516 | 1.41 |
| 35280 | 1.36 |
| 37044 | 1.21 |
| 38808 | 1.15 |
| 40572 | 1 |
| 42336 | 1.72 |
| 44100 | 1.51 |

TABLE 14

| Freq | Amplitude |
|-------|-----------|
| 0 | 1.61 |
| 3392 | 1 |
| 6784 | 1.19 |
| 10176 | 1.41 |
| 13568 | 1.72 |
| 16960 | 2.49 |
| 20352 | 2.27 |
| 23744 | 2.27 |
| 27136 | 2.49 |
| 30528 | 1.72 |
| 33920 | 1.41 |
| 37312 | 1.19 |
| 40704 | 1 |

TABLE 15

| Freq | Amplitude |
|-------|-----------|
| 0 | 1.7 |
| 2310 | 1.6 |
| 4620 | 1 |
| 6930 | 1.2 |
| 9240 | 1.3 |
| 11550 | 1.4 |
| 13860 | 1.5 |
| 16170 | 1.6 |
| 18480 | 2.1 |
| 20790 | 2 |
| 23100 | 2 |
| 25410 | 2.1 |
| 27720 | 1.6 |
| 30030 | 1.5 |
| 32340 | 1.4 |
| 34650 | 1.3 |
| 36960 | 1.2 |
| 39270 | 1 |
| 41580 | 1.6 |

TABLE 16

| Freq | Amplitude |
|-------|-----------|
| 0 | 1.72 |
| 4009 | 1 |
| 8018 | 1.35 |
| 12027 | 1.8 |
| 16036 | 3 |
| 20045 | 3.15 |
| 24054 | 3.15 |
| 28063 | 3 |
| 32072 | 1.8 |
| 36081 | 1.35 |
| 40090 | 1 |

TABLE 17

| Freq | Amplitude |
|-------|-----------|
| 0 | 1.72 |
| 4009 | 1 |
| 8018 | 1.35 |
| 12027 | 2 |
| 16036 | 3.5 |
| 20045 | 3.15 |
| 24054 | 3.15 |
| 28063 | 3.5 |
| 32072 | 2 |
| 36081 | 1.35 |
| 40090 | 1 |

TABLE 18

| Freq | Amplitude |
|-------|-----------|
| 0 | 2 |
| 4009 | 1 |
| 8018 | 1.35 |
| 12027 | 2 |
| 16036 | 4 |
| 20045 | 3.15 |
| 24054 | 3.15 |
| 28063 | 4 |
| 32072 | 2 |
| 36081 | 1.35 |
| 40090 | 1 |

TABLE 19

| Freq | Amplitude |
|-------|-----------|
| 0 | 2 |
| 4009 | 1 |
| 8018 | 1.35 |
| 12027 | 1.65 |
| 16036 | 2.59 |
| 20045 | 3.5 |
| 24054 | 3.5 |
| 28063 | 2.59 |
| 32072 | 1.65 |
| 36081 | 1.35 |
| 40090 | 1 |

TABLE 20

| Freq | Amplitude | Time Coefficient | Phase |
|-------|-----------|------------------|--------|
| 0 | 2.5 | 2.8684 | 0 |
| 2321 | 2.5 | 0.3105 | 3.1416 |
| 4642 | 2 | 0.0385 | 3.1416 |
| 6963 | 2.3 | 0.1654 | 0 |
| 9284 | 2.5 | 0.0134 | 0 |
| 11605 | 2.9 | 0.0268 | 3.1416 |
| 13926 | 3.5 | 0.1568 | 0 |
| 16247 | 4 | 0.1367 | 3.1416 |
| 18568 | 2.6 | 0.0653 | 0 |
| 20889 | 3.5 | 0.0727 | 3.1416 |
| 23210 | 3.5 | 0.0727 | 3.1416 |
| 25531 | 2.6 | 0.0653 | 0 |
| 27852 | 4 | 0.1367 | 3.1416 |
| 30173 | 3.5 | 0.1568 | 0 |
| 32494 | 2.9 | 0.0268 | 3.1416 |
| 34815 | 2.5 | 0.0134 | 0 |
| 37136 | 2.3 | 0.1654 | 0 |
| 39457 | 2 | 0.0385 | 3.1416 |
| 41778 | 2.5 | 0.3105 | 3.1416 |

Phase means phase of the time coefficient. All Frequency phases are zero.

TABLE 21

| Freq | Amplitude |
|-------|-----------|
| 0 | 2.5 |
| 4009 | 1 |
| 8018 | 2.5 |
| 12027 | 2.85 |
| 16036 | 4 |
| 20045 | 3.5 |
| 24054 | 3.5 |
| 28063 | 4 |
| 32072 | 2.85 |
| 36081 | 2.5 |
| 40090 | 1 |

TABLE 22

| Freq | Amplitude |
|-------|-----------|
| 0 | 2.5 |
| 4009 | 1 |
| 8018 | 2.5 |
| 12027 | 2.85 |
| 16036 | 4.5 |
| 20045 | 3.5 |
| 24054 | 3.5 |
| 28063 | 4.5 |
| 32072 | 2.85 |
| 36081 | 2.5 |
| 40090 | 1 |

TABLE 23

| Freq | Amplitude |
|-------|-----------|
| 0 | 1.9 |
| 2321 | 1.8 |
| 4642 | 1 |
| 6963 | 1.2 |
| 9284 | 1.3 |
| 11605 | 1.4 |
| 13926 | 1.5 |
| 16247 | 1.6 |
| 18568 | 2.1 |
| 20889 | 2 |
| 23210 | 2 |
| 25531 | 2.1 |
| 27852 | 1.6 |
| 30173 | 1.5 |
| 32494 | 1.4 |
| 34815 | 1.3 |
| 37136 | 1.2 |
| 39457 | 1 |
| 41778 | 1.8 |

TABLE 24

| Freq | Amplitude |
|-------|-----------|
| 0 | 2.5 |
| 4009 | 1.5 |
| 8018 | 2.5 |
| 12027 | 2.85 |
| 16036 | 4 |
| 20045 | 3.5 |
| 24054 | 3.5 |
| 28063 | 4 |
| 32072 | 2.85 |
| 36081 | 2.5 |
| 40090 | 1.5 |

TABLE 25

| Frequency | Amplitude |
|-----------|-----------|
| 0 | 3 |
| 4009 | 2 |
| 8018 | 2.5 |
| 12027 | 2.85 |
| 16036 | 4 |
| 20045 | 3.5 |
| 24054 | 3.5 |
| 28063 | 4 |
| 32072 | 2.85 |
| 36081 | 2.5 |
| 40090 | 2 |

TABLE 26

| Frequency | Amplitude |
|-----------|-----------|
| 0 | 3 |
| 4009 | 1.5 |
| 8018 | 2.5 |
| 12027 | 2.85 |
| 16036 | 4 |
| 20045 | 3.5 |
| 24054 | 3.5 |
| 28063 | 4 |
| 32072 | 2.85 |
| 36081 | 2.5 |
| 40090 | 1.5 |

TABLE 27

| Freq | Amplitude |
|-------|-----------|
| 0 | 1.9 |
| 2321 | 1.8 |
| 4642 | 1 |
| 6963 | 1.5 |
| 9284 | 1.6 |
| 11605 | 1.7 |
| 13926 | 1.8 |
| 16247 | 1.9 |
| 18568 | 2.2 |
| 20889 | 2 |
| 23210 | 2 |
| 25531 | 2.2 |
| 27852 | 1.9 |
| 30173 | 1.8 |
| 32494 | 1.7 |
| 34815 | 1.6 |
| 37136 | 1.5 |
| 39457 | 1 |
| 41778 | 1.8 |

TABLE 28

| Freq | Amplitude | Time Coefficient | Phase |
|-------|-----------|------------------|--------|
| 0 | 2.5 | 2.8636 | 0 |
| 4009 | 1 | 0.6084 | 3.1416 |
| 8018 | 2.5 | 0.1736 | 3.1416 |
| 12027 | 3.5 | 0.2247 | 0 |
| 16036 | 4 | 0.1454 | 0 |
| 20045 | 3.5 | 0.23 | 0 |
| 24054 | 3.5 | 0.23 | 0 |
| 28063 | 4 | 0.1454 | 0 |
| 32072 | 3.5 | 0.2247 | 0 |
| 36081 | 2.5 | 0.1736 | 3.1416 |
| 40090 | 1 | 0.6084 | 3.1416 |

Phase means phase of the time coefficient. All Frequency phases are zero.

TABLE 29

| Freq | Amplitude |
|-------|-----------|
| 0 | 2 |
| 1131 | 1.66 |
| 2262 | 1.33 |
| 3393 | 1 |
| 4524 | 1 |
| 5655 | 1.3 |
| 6786 | 1.4 |
| 7917 | 1.5 |
| 9048 | 1.6 |
| 10179 | 1.7 |
| 11310 | 1.8 |
| 12441 | 1.9 |
| 13572 | 2 |
| 14703 | 2.1 |
| 15834 | 2.2 |
| 16965 | 2.4 |
| 18096 | 2.5 |
| 19227 | 2.6 |
| 20358 | 2.7 |
| 21489 | 2.8 |
| 22620 | 2.7 |
| 23751 | 2.6 |
| 24882 | 2.5 |
| 26013 | 2.4 |
| 27144 | 2.3 |
| 28275 | 2.2 |
| 29406 | 2.1 |
| 30537 | 2 |
| 31668 | 1.9 |
| 32799 | 1.8 |
| 33930 | 1.7 |
| 35061 | 1.6 |
| 36192 | 1.5 |
| 37323 | 1.4 |
| 38454 | 1.3 |
| 39585 | 1 |
| 40716 | 1 |
| 41847 | 1.33 |
| 42978 | 1.66 |

TABLE 30

| Frequency | Amplitude |
|-----------|-----------|
| 0 | 2.5 |
| 4009 | 2 |
| 8018 | 2.5 |
| 12027 | 2.85 |
| 16036 | 4.5 |
| 20045 | 3.5 |
| 24054 | 3.5 |
| 28063 | 4.5 |
| 32072 | 2.85 |
| 36081 | 2.5 |
| 40090 | 2 |

TABLE 31

| Frequency | Amplitude |
|-----------|-----------|
| 0 | 2.5 |
| 2321 | 2.6 |
| 4642 | 2.15 |
| 6963 | 2.4 |
| 9284 | 2.6 |
| 11605 | 2.9 |
| 13926 | 3.78 |
| 16247 | 4.6 |
| 18568 | 2.6 |
| 20889 | 4 |
| 23210 | 4 |
| 25531 | 2.6 |
| 27852 | 4.6 |
| 30173 | 3.78 |
| 32494 | 2.9 |
| 34815 | 2.6 |
| 37136 | 2.4 |
| 39457 | 2.15 |
| 41778 | 2.5 |

TABLE 32

| Freq | Amplitude |
|-------|-----------|
| 0 | 1.6 |
| 2321 | 1.5 |
| 4642 | 1 |
| 6963 | 1.12 |
| 9284 | 1.2 |
| 11605 | 1.3 |
| 13926 | 1.4 |
| 16247 | 1.6 |
| 18568 | 2 |
| 20889 | 1.9 |
| 23210 | 1.9 |
| 25531 | 2 |
| 27852 | 1.6 |
| 30173 | 1.4 |
| 32494 | 1.3 |
| 34815 | 1.2 |
| 37136 | 1.12 |
| 39457 | 1 |
| 41778 | 1.5 |

TABLE 33

| Freq | Amplitude | Time Coefficients | Phase |
|-------|-----------|-------------------|--------|
| 0 | 3 | 1.2759 | 0 |
| 1521 | 2 | 0.2615 | 0 |
| 3042 | 3 | 0.2209 | 0 |
| 4563 | 1 | 0.1608 | 0 |
| 6084 | 1 | 0.0913 | 0 |
| 7605 | 1 | 0.0239 | 0 |
| 9126 | 1 | 0.0308 | 3.1416 |
| 10647 | 1 | 0.0644 | 3.1416 |
| 12168 | 1 | 0.0729 | 3.1416 |
| 13689 | 1 | 0.0567 | 3.1416 |
| 15210 | 1 | 0.0208 | 3.1416 |
| 16731 | 1 | 0.0264 | 0 |
| 18252 | 1 | 0.0745 | 0 |
| 19773 | 1 | 0.1134 | 0 |
| 21294 | 1 | 0.1351 | 0 |
| 22815 | 1 | 0.1351 | 0 |
| 24336 | 1 | 0.1134 | 0 |
| 25857 | 1 | 0.0745 | 0 |
| 27378 | 1 | 0.0264 | 0 |
| 28899 | 1 | 0.0208 | 3.1416 |
| 30420 | 1 | 0.0567 | 3.1416 |
| 31941 | 1 | 0.0729 | 3.1416 |
| 33462 | 1 | 0.0644 | 3.1416 |
| 34983 | 1 | 0.0308 | 3.1416 |
| 36504 | 1 | 0.0239 | 0 |
| 38025 | 1 | 0.0913 | 0 |
| 39546 | 1 | 0.1608 | 0 |
| 41067 | 3 | 0.2209 | 0 |
| 42588 | 2 | 0.2615 | 0 |

Phase means phase of the time coefficient. All Frequency phases are zero.

TABLE 34

| Freq | Amplitude | Time Coefficient | Phase |
|-------|-----------|------------------|--------|
| 0 | 2 | 1.9487 | 0 |
| 1131 | 1 | 0.0506 | 3.1416 |
| 2262 | 2 | 0.0486 | 3.1416 |
| 3393 | 2 | 0.0454 | 3.1416 |
| 4524 | 2 | 0.041 | 3.1416 |
| 5655 | 2 | 0.0355 | 3.1416 |
| 6786 | 2 | 0.0291 | 3.1416 |
| 7917 | 2 | 0.022 | 3.1416 |
| 9048 | 2 | 0.0143 | 3.1416 |
| 10179 | 2 | 0.0062 | 3.1416 |
| 11310 | 2 | 0.0021 | 0 |
| 12441 | 2 | 0.0103 | 0 |
| 13572 | 2 | 0.0182 | 0 |
| 14703 | 2 | 0.0256 | 0 |
| 15834 | 2 | 0.0324 | 0 |
| 16965 | 2 | 0.0384 | 0 |
| 18096 | 2 | 0.0433 | 0 |
| 19227 | 2 | 0.0472 | 0 |
| 20358 | 2 | 0.0498 | 0 |
| 21489 | 2 | 0.0511 | 0 |
| 22620 | 2 | 0.0511 | 0 |
| 23751 | 2 | 0.0498 | 0 |
| 24882 | 2 | 0.0472 | 0 |
| 26013 | 2 | 0.0433 | 0 |
| 27144 | 2 | 0.0384 | 0 |
| 28275 | 2 | 0.0324 | 0 |
| 29406 | 2 | 0.0256 | 0 |
| 30537 | 2 | 0.0182 | 0 |
| 31668 | 2 | 0.0103 | 0 |
| 32799 | 2 | 0.0021 | 0 |
| 33930 | 2 | 0.0062 | 3.1416 |
| 35061 | 2 | 0.0143 | 3.1416 |
| 36192 | 2 | 0.022 | 3.1416 |
| 37323 | 2 | 0.0291 | 3.1416 |
| 38454 | 2 | 0.0355 | 3.1416 |
| 39585 | 2 | 0.041 | 3.1416 |
| 40716 | 2 | 0.0454 | 3.1416 |
| 41847 | 2 | 0.0486 | 3.1416 |
| 42978 | 1 | 0.0506 | 3.1416 |

Phase means phase of the time coefficient. All Frequency phases are zero.